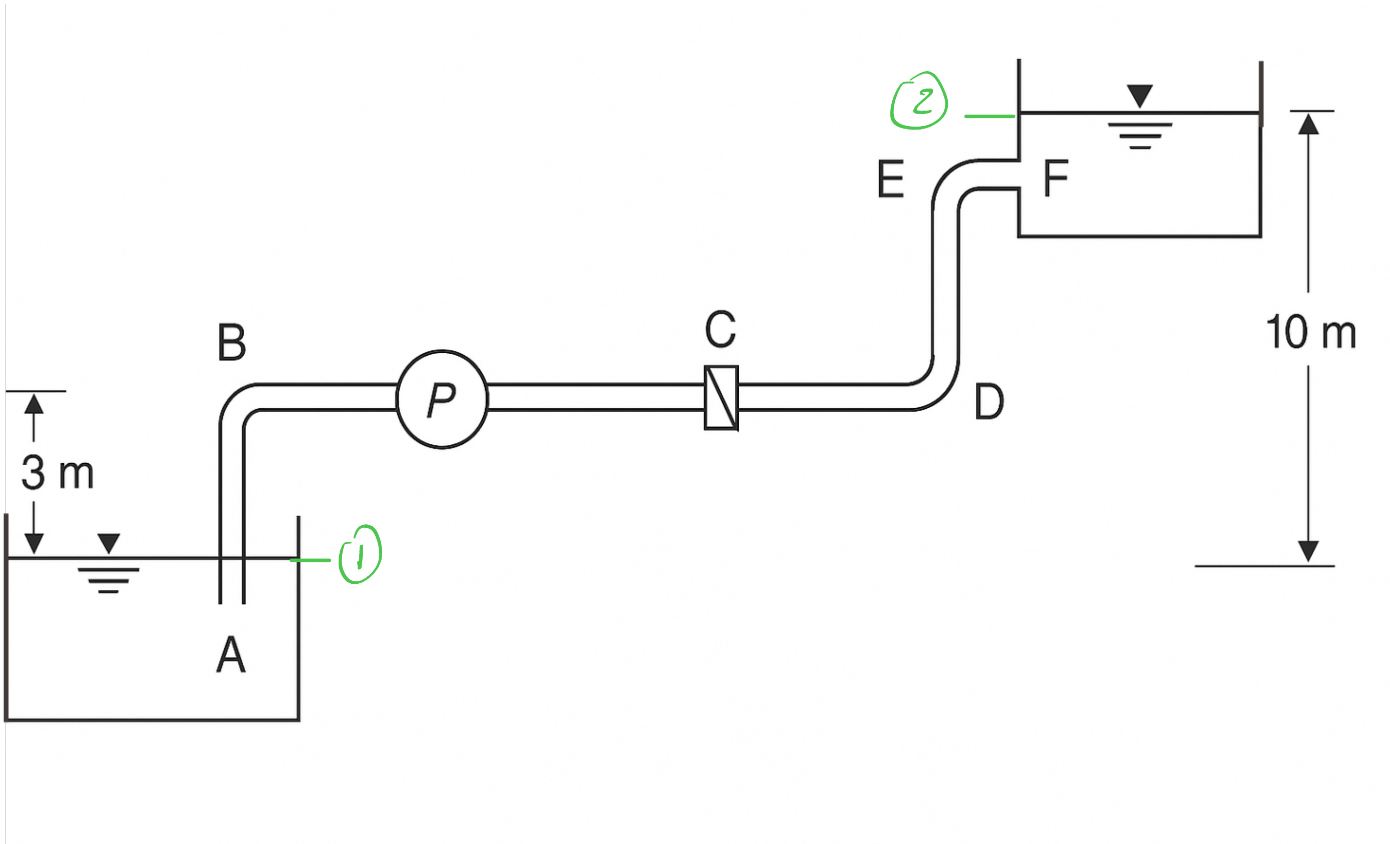
Example on Pumps



Water ($v = 1.00e\text{-}6\ m^2/s$, $\gamma = 9789\ N/m^3$) is being pumped from reservoir A to reservoir F through a 30-m-long PVC pipe ($\epsilon = 0$) of diameter 150 mm.

There is an open gate valve ($k_{gv} = 0.2$) located at C; 90º bends (threaded) located at B, D, and E ($k_{bend} = 0.9$); $k_{entr} = 1.0$; $k_{exit} = 1.0$; and the pump performance curve is given by:

$$h_p = 20 - 4713Q^2$$

where h_p is the head added by the pump in meters and Q is the flowrate in m^3/s. The pump manufacturer has provided a $NPSH_R = 2.0$ m.

1. Write the energy equation between the upper and lower reservoirs in terms of h_{sys}, f, and Q
2. Calculate the flowrate and velocity in the pipe. Use the C&W equation to estimate f.
3. What is the net positive suction head if the suction pipe has a length of 10 m? Will cavitation occur in the pump? It's 20 degree Celsius outside. Explain your answer.

1) We start by writing the energy equation for the system between the free surfaces of reservoir A and F (points 1 and 2).

$$\cancel{\frac{P_1}{\gamma}} + z_1 + \cancel{\frac{V_1^2}{2g}} + h_p = \cancel{\frac{P_2}{\gamma}} + z_2 + \cancel{\frac{V_2^2}{2g}} + h_{L_{TOT}}$$

$$h_{L_{TOT}} = \frac{f}{D} L \frac{V^2}{g} + \Sigma K \frac{V^2}{2g}$$

$$= \frac{V^2}{2g}\left(\frac{fL}{D} + \Sigma K\right)$$

$$h_p = (z_2 - z_1) + \frac{V^2}{2g}\left(\frac{fL}{D} + \Sigma K\right)$$

$$h_p = \underbrace{(z_2 - z_1) + \frac{Q^2}{2gA^2}\left(\frac{fL}{D} + \Sigma K\right)}_{h_{sys}}$$

$h_{sys} \Rightarrow h_p = h_{sys}$

$\rightarrow z_2 - z_1 = 10\ m$

$\rightarrow \frac{fL}{D} = f \cdot \left(\frac{30\ m}{0.15\ m}\right) = 200f$

$\rightarrow \Sigma K = 1\ [\text{entr}] + 3\cdot(0.9)\ [90°\ \text{elbows}] + 0.2\ [\text{gate valve}] + 1\ [\text{exit}]$

$= 4.9$

$$\longrightarrow \quad \frac{Q^2}{2gA^2} = \frac{Q^2}{2 \cdot (9.81\ m/s^2)(0.01767\ m^2)^2} \qquad A = \frac{\pi}{4}(0.15)^2 = 0.01767\ m^2$$

$$= 163\ Q^2$$

Put everything together:

$$\Longrightarrow \quad h_{sys} = 10 + 163 Q^2 (200 f + 4.9)$$

2) find v and Q. We start by setting h_p = h_sys

$$h_p = h_{sys}$$

$$20 - 4713 Q^2 = 10 + 163 Q^2 (200 f + 4.9)$$

$$10 = Q^2 [163 (200 f + 4.9) + 4713]$$

$$Q = \left\{ \frac{10}{[163 (200 f + 4.9) + 4713} \right\}^{1/2}$$

$\longrightarrow$ COLEBROOK & WHITE (C&W)

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\cancel{\frac{1}{3.71} \frac{\varepsilon}{D}} + \frac{2.51}{Re\sqrt{f}} \right)$$

for PVC $\Longrightarrow \varepsilon = 0$

$$\Longrightarrow f = \left[-2 \log_{10} \left(\frac{2.51}{Re\sqrt{f}} \right) \right]^{-2}$$

with $Re = \frac{VD}{\nu} = \frac{4Q}{\pi \nu D}$

To solve for Q, we select a starting value for f, then we solve the Q expression found earlier, use Q in C & W to find new value for f and REITERATE if not correct.

ITERATION	f	Q (m^3/s)	V (m/s)	f_{new}
I	0.015	0.0408	2.31	0.014
II	0.014	0.0409	2.32	0.014 ✓ converged

these are the final values.

3) Will cavitation occur? We need to calculate the Net Poistive Suction Head (with respect to the suction side of the pump) and compare it to the NPSH_R given by the pump manufacturer.

Let's apply energy balance between reservoir A and the suction side of the pump.

$$\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{P_s}{\gamma} + z_s + \frac{V_s^2}{2g} + h_{L,1\to s}$$

$$\Downarrow$$

$$NPSH_A = \left(\frac{P_s}{\gamma} + z_s + \frac{V_s^2}{2g}\right) - \left(\frac{P_v}{\gamma} + z_s\right)$$

$$NPSH_A = \left(\frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - h_L\right) - \left(\frac{P_v}{\gamma} + z_s\right)$$

$P_1 = P_{ATM}$!

$V_1 = 0$

$$NPSH_A = \frac{P_{ATM} - P_V}{\gamma} - (z_S - z_1) - h_L$$

$$\rightarrow h_L = \frac{V^2}{2g}\left(\frac{fL}{D} + \Sigma K\right) \qquad L_{1\to S} = 10\,m$$

$$= \frac{Q^2}{2gA^2}\left[\frac{0.014\,(10\,m)}{0.15\,m} + 1\,[\text{entr}] + 0.9\,[90°\text{ elbow}]\right]$$

$$Q = 0.0409\ m^3/s$$

$$A = 0.01767\ m^2$$

$$= \frac{(0.0409\ m^3/s)^2}{2(9.81\ m/s^2)(0.01767\ m^2)^2} \cdot 2.833$$

$$= 0.774\ m$$

$$\rightarrow P_{ATM} = 101.3 \times 10^3\ \frac{N}{m^2}$$

$$\rightarrow P_V\ @\ 20°C\ ?$$

Physical Properties of Water (SI Units)[a]

Temperature (°C)	Density, ρ (kg/m^3)	Specific Weight[b], γ (kN/m^3)	Dynamic Viscosity, μ (N·s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension[c], σ (N/m)	Vapor Pressure, p_v [N/m^2(abs)]	Speed of Sound[d], c (m/s)
0	999.9	9.806	1.787 E − 3	1.787 E − 6	7.56 E − 2	6.105 E + 2	1403
5	1000.0	9.807	1.519 E − 3	1.519 E − 6	7.49 E − 2	8.722 E + 2	1427
10	999.7	9.804	1.307 E − 3	1.307 E − 6	7.42 E − 2	1.228 E + 3	1447
20	998.2	9.789	1.002 E − 3	1.004 E − 6	7.28 E − 2	2.338 E + 3	1481
30	995.7	9.765	7.975 E − 4	8.009 E − 7	7.12 E − 2	4.243 E + 3	1507
40	992.2	9.731	6.529 E − 4	6.580 E − 7	6.96 E − 2	7.376 E + 3	1526
50	988.1	9.690	5.468 E − 4	5.534 E − 7	6.79 E − 2	1.233 E + 4	1541
60	983.2	9.642	4.665 E − 4	4.745 E − 7	6.62 E − 2	1.992 E + 4	1552
70	977.8	9.589	4.042 E − 4	4.134 E − 7	6.44 E − 2	3.116 E + 4	1555
80	971.8	9.530	3.547 E − 4	3.650 E − 7	6.26 E − 2	4.734 E + 4	1555
90	965.3	9.467	3.147 E − 4	3.260 E − 7	6.08 E − 2	7.010 E + 4	1550
100	958.4	9.399	2.818 E − 4	2.940 E − 7	5.89 E − 2	1.013 E + 5	1543

[a]Based on data from *Handbook of Chemistry and Physics*, 69th Ed., CRC Press, 1988.

[b]Density and specific weight are related through the equation $\gamma = \rho g$. For this table, $g = 9.807$ m/s^2.

[c]In contact with air.

[d]Based on data from R. D. Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Co., Inc., New York, 1984.

$$P_v = 2338 \ \frac{N}{m^2} \ \text{(absolute!)}$$

$$\longrightarrow \ z_s - z_1 = 3\,m$$

$$\Longrightarrow \ NPSH_A = \frac{(101.3 \cdot 10^3 - 2.338 \cdot 10^3) \ N/m^2}{9789 \ N/m^3} - 3\,m - 0.744\,m$$

$$= 6.37\,m$$

Since NPSH_A = 6.37 m is greater than the NPSH_R = 2 m, cavitation will not occur!